THE 42nd—14th INTERNATIONAL—RUDOLF ORTVAY PROBLEM SOLVING CONTEST IN PHYSICS
28 October—7 November 2010

1. Dr. Absoluto Zero, the widely known philantropic dictator of Gummy Coast, a small equatorial country, has decided to help his people in the gummy harvest, which is a tiring job excersised for many centuries. So far, people would climb on the gummy trees or on ladders and get the gummyberries (fruits of the gummy tree) beaten off from the branches by sticks.

   The Biotechnological Research Center of Gummy Coast (named after dr. Absoluto Zero) has developed a strain of gummyberries with highly increased elasticity and ability to bounce.

   The proponents of the Self Harvesting Gummyberry Project (including the Prime Physicist of the country, decorated by the National Coriolis Order for solving a former Ortvay problem) are assuming that from now on, it will be enough to get a single berry released from the end of the line of gummy trees. This would bounce back up from the ground, would hit and shake the branches of the neighbouring tree, release further berries, and by bouncing induce an avalanche to self-harvest even more gummyberries.

   (The standardized gummy palm has a leafage with a radius of exactly 137 cm, the mature berries are uniformly distributed among the leafage. The plantation, by the order of Absoluto Zero, is organized in a triangular grid, where the grid points where the trees are planted are 3 meters from each other. The leaves of a gummy palm are oblong, their edges are prickly, the front surface is bright green, their overleaf is matt dark green and slightly sammy.)

   Unfortunately – causing big sorrow for the dictator – the ceremonial inaguration of the first Self Harvesting Gummyplantation needed to be postponed, as the trees turned out not to be tall enough so that the chain reaction which would trigger the self harvesting of the neighbouring trees would start.

   Let us calculate, how high the gummy palm trees – with straight tree trunks – should grow so that the automatic harvesting would start. This issue is evidently of interest from the international stock markets of gummy production.

   (Gyula Dávid)

2. The space shuttle entering the Earth's atmosphere from a height of 100 km can glide 450 km horizontal distance. Our next goal is the soft landing on a spherical asteroid's solid surface with radius of 100 km. Assume the space shuttle can glide with the same angle in the atmosphere of the asteroid as in the atmosphere of the Earth. What distance could the space shuttle glide (measured on the asteroid's surface) in the asteroid's atmosphere starting from a height of 100 km?

   (Béla Ráczkevi)

3. The yo-yo game reserves some surprises in spite of being simple. Consider the following model: two equal parallel disks of radius R, and in between a cylindrical axis with radius r_0 and length $d \ll R$. The rotating axis is perpendicular to the disks.

   A thread is wound around the axis. The thickness of the thread is d, and the length is l. The moment of inertia of the yo-yo is, with a good approximation, $\theta = \frac{1}{2}mR^2$.

   We release the yo-yo to move in vertical direction, keeping the end of the thread in our hand fixed.

   Determine the acceleration $a(x)$ of the mid-point of the yo-yo and its velocity $v(x)$, as a function of the length x of the thread which unwounded from the shaft.

   Plot these functions with freely chosen parameters against x in the interval of $[0, l]$!

   Determine the lenght of the unwounded thread x as a function of time! How long does it take the whole length l to unwound? Plot the function $t(x)$ with the above chosen parameters!

   At which value of x does the speed of the center of mass get maximal? Let $\alpha = r_0/R$, and plot the maximum value of the velocity as a function of α, as well as the total time needed for the thread to unwound! Is there anything interesting to find? During the calculations, the friction and the mass of the thread is negligible, and assume a continuous, non slipping rolling of the yo-yo!

   (Gábor Homa)

4. An astronomer wanted to know the geographic coordinates of his house. Since he had no GPS he observed the Sun rise. He knew he could observe time much more accurately than position. Therefore he observed only the time of the Sun rise. He observed the first light of the Sun at the following times in GMT:

Date	28/10/2010	15/11/2010	24/11/2010	25/11/2010	10/12/2010	11/12/2010
Time	8:23:31 $\pm$5 sec	8:50:16 $\pm$10 sec	9:04:55 $\pm$15 sec	9:05:25 $\pm$10 sec	9:22:15 $\pm$5 sec	9:23:02 $\pm$3 sec

28/01/2011	07/02/2011	15/02/2011	09/03/2011	04/04/2011	11/04/2011	10/ 05/2011
9:17:25 $\pm$10 sec	9:03:25 $\pm$5 sec	8:51 $\pm$30 sec	8:11:32 $\pm$10 sec	7:20:05 $\pm$15 sec	7:06:45 $\pm$10 sec	6:20: $\pm$2 min

Where exactly is his house located?

(István Horváth)

5. A small solid ball of radius r is pressed against the inner side of a fixed long cylindrical tube with a vertical axis and of inner radius R, and then released with a horizontal speed v_0 which is large enough so that the ball starts to move in an interesting fashion inside the tube, all the time touching the innner wall of the tube. Describe motion of of the ball's center! (Let us assume that the friction is high enough so that the ball does not slip during its motion.)

(Máté Vigh)

6. The new sensation of Ice Circus is the rotating skating rink! This is a huge, horizontal ice disk which rotates at constant angular velocity.

   Study the movement of the skaters on the rotating ice! The ideal skate is a thin blade, which can move only in the direction of its line, without friction. The movements which would occur in the direction perpendicular to the blade are prevented by the forces acting between the ice and the skate. Calculate the possible trajectories of the movements of the skaters, and plot the curves of their path! Which are primary parameters to determine which curve type will be actually realized during the sliding?

(Gyula Dávid and József Cserti)

7. A small metal sphere with charge Q and radius R is placed into a cloud chamber. Close to the conductive sphere, an oppositely charged classical particle with $-q$ charge is moving, which experiences a drag force proportional to the square of its speed, $\mathbf{F}_{\rm drag} = -C|\mathbf{v}|\mathbf{v}$, which is relatively small compared to the electrostatic force. Here C is a constant since the medium in the chamber has homogeneous density. Gravity and induced charge on the sphere can be neglected for the rest of the calculations.
   a) Study the movement of the charged particle, if its trajectory is approximately circular in the beginning. How does the distance to the metal sphere, and the speed of the particle change as a function of time? What is the tangential component of the total force acting on the particle? Prepare an explanatory figure indicating the force vectors.
   b) Study the movement of the particle if the trajectory is similar to an elongated ellipse in the beginning. How does the angular momentum (with respect to the middle of the sphere) change as a function of the arc length along the trajectory? How does the shape of the trajectory change: is it getting more elongated, or gets more circular? Why? Let us try to give an explanation which would be simple enough to be understandable for secondary school students.

(Gyula Honyek and Máté Vigh)

8. An irregularly shaped, 'potato-like' asteroid is moving in a slightly inhomogeneous, central $V(r)$ potential field $V(r)$. Calculate the tidal torque acting on the asteroid and express the result with the tensor of moment of inertia measured in the center of mass of the body, and the data from the potential! Specialize the problem for spheroid-shaped (oblate or prolate) asteroids revolving around the Sun in a circular orbit, and give the formula for the torque as a function of the revolution time, the moments of inertia along the principal axes, and the angle between the symmetry axis of the body and the normal vector of the orbital plane!

(Gyula Dávid)

9. A 'barrel' has the shape of two truncated cones, adjoined at their larger base plane. The barrel is placed on a pair of very thin, sloped parallel rails, and it is let to roll. The drag force is proportional to the velocity of the center of mass. At which speed does the barrel's motion become unstable? Can we design a body shape, for which the stable rolling is ensured only *above* a certain speed?

(András Bodor)

10. The planet Trundler circulates around its sun on a circular orbit, and the axis of its own revolution is laying in the orbital plane. Literally, three days on the planet is a year. Which is the trajectory of the sun of the planet seen from the planet surface a) from the poles b) on the planet's equator c) for some observer on the intermediate altitudes? What are the seasons like on Trundler, if the primary planetological parameters (distance to the sun, incoming radiation, average surface temperature and pressure, composition of the atmosphere and hydrosphere) and the length of the year is the same as those on Earth?

(Gyula Dávid)

11. There are a number of transmisson satellites circling around a spherical planet on the equatorial synchronous orbit, broadcasting harshly advertisements and pop music. The oceans of the planet are long ago dried out, the complete surface of the planet is inhabited in a uniform fashion. The masters of the broadcast system are feeling disturbed about the fact that the ads do not get to all inhabitants, therefore they ask the scientists to determine which fraction of the population is unable to recieve the transmission from the synchronous orbit. Let us express this fraction using the effective gravitational acceleration on the poles and on the equator.

(Gyula Dávid)

12. A supersonic fighter plane starts from rest with constant acceleration a along a straight line. In this case, in certain places at certain times, one can hear a 'super-sonic boom' instead of the usual sonic boom experienced in flight at constant supersonic speed.

    Determine in an analytic fashion the position of the 'super-sonic boom' relative to the fighter aircraft as a function of time from departure! Calculate and plot the envelope curve of the sound waves produced by the aircraft! What is the indication on the enveloping curve for the 'super-sonic boom'?

(András Bodor)

13. The civilization of Atlantis, as it is known, was flourishing 9000 years ago underseas, down in the Falkland Sound, the channel between the Falkland Islands. Their traces have been discovered by Lee Ben Canal and A. C. Boowar from the University of Farewell. The researchers have now found the mechanical control unit of the main telescope in the underwater Atlantis Observatory. This masterpiece of the Atlantis fine mechanics industry was responsible to keep the underseas telescope following the direction of the apparent stars, as seen on the sky from underseas. Let us calculate what are the trajectories of the stars at different declination angle during one night on the sky of Atlantis as seen from below the water, and also let us draw down these curves!

(Gyula Dávid)

14. If a charged particle moves faster than the speed of light in a dielectric material along a straight line, then it will emit Cherenkov radiation in a conical shape. Consider a charged particle which moves along a circle at constant superluminal speed. Determine the shape of the wave front in the plane of the circle. Which known curves can describe the shape of the wave front in different regions of the plane of the circle?

    Plot the wave front at different values of the ratio v/c, where v is the speed of the particle, and c is the speed of light in the material.

(Máté Vigh and András Bodor)

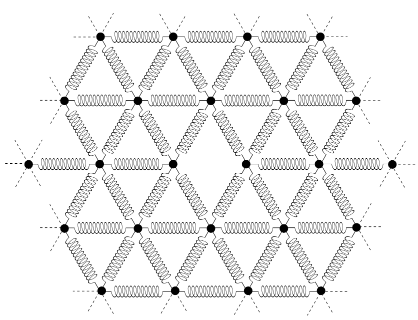
15. Determine the frequency spectrum of a linear chain, which is made of N equivalent balls and N equivalent springs connecting them, when the boundary condition, instead of the usual peroidic one, is given as:

$$u_N(t) = u_0(t + \tau)$$

    here τ is a fixed delay time. Study the orthogonality of the normal modes!

(Géza Tichy, co-founder of the Ortvay contest)

16. Point like balls of mass m are located at the corners of a perfect infinite triangular lattice. They are connected by perfect springs of spring constant D, except two nearest neighbor balls, where the spring is missing between them (see the figure). The balls can move only within the plane of the triangular lattice. How do the frequencies of the lattice vibrations change with respect to the perfect lattice (when the spring is not missing)? Determine the frequencies of the lattice vibrations numerically!



(József Cserti and Géza Tichy, co-founder of the Ortvay contest)

17. Within the framework of Hooke's law calculate the elastic deformation of a homogenous sphere of radius R which rotates along its symmetry axis with angular velocity ω.
Estimate the deformation at the poles and at the equator for a copper sphere of radius $R = 1$ m for $f = 10.000$ rpm.

(József Cserti)

18. A gas bubble is in equilibrium at the bottom of a vessel filled with liquid. What is its possible shape and size? The specific weight of the liquid and the gas are γ_1 and γ_2, the surface tension is α, the contact angle (the angle between the liquid–solid and liquid–gas surfaces) is ϑ. (For the sake of definiteness let's have $\Delta\gamma = \gamma_1 - \gamma_2 = 10^4\,\mathrm{N/m^3}$, $\alpha = 0{,}07\,\mathrm{J/m^2}$ and $\vartheta = 5°$.)

(Ferenc Woynarovich)

19. One can blow air out with higher intensity than to suck air in. In relation to this fact, let us study the symmetry properties of the Navier–Stokes equation!

(Géza Tichy, co-founder of the Ortvay contest)

20. Ultrasonic waves are created in water, using a thin, disk shaped piezo-electric crystal which is resonated electrically. We are studying the longitudinal waves produced on one side of the crystal, along the symmetry axis perpendicular to the plane of the disk.

One can find surprisingly, that the intensity at 6 cm distance from the disk is zero. At greater distances, the intensity becomes measurable again, and, after reaching a maximum, will decrease continuously.

a) What intensity would we measure at distances smaller than 6 cm?

b) What is the diameter of the disk?

The resonant frequency of the disk is 5 MHz, the speed of sound in the water under study is 1500 m/s.

(Gyula Radnai)

21. Let us try to design an escapement system for pendulum clocks, which does not produce the characteristic clicking sound!

One can solve the problem using permanent magnets, as relatively strong magnets are now readily available. One can think of a structure analogous with the classic arrangement, where the rotation is blocked by not the teeth, but magnets getting close to each other.

a) How such an escapement system would look like? (Consider the necessity of recovering the energy loss of the pendulum, which otherwise stops swinging!)

b) How much energy can such a mechanism transfer from the wheel to the pendulum to counteract the friction on the pendulum?

c) Under which conditions such a motion is prevented from being chaotic?

(György Radnóczi Jr.)

22. An insulating ball with homogeneous charge and mass density, with total charge Q and total mass m is placed in magnetic field. The ball is spinned with angular speed $\boldsymbol{\omega}_0$ and pushed to move with velocity $\mathbf{v}_0$
a) Write down the equations of the movement and rotation for the ball in its simplest form. Consider the magnetic field's spatial variation only up to the zeroth and first order terms (homogeneous field and field gradient) in the series expansion.
b) Solve the equations in homogeneous field, that is, let $\mathbf{B}(\mathbf{r}) = (0, 0, B_0)$, and for simplicity, $\mathbf{v}_0 = (v_0, 0, 0)$, and $\boldsymbol{\omega}_0$ is arbitrary!
c) Solve the equations in case of magnetic field with constant gradient, that is, let $\mathbf{B}(\mathbf{r}) = (0, 0, B_0 + \alpha y)$, $\mathbf{v}_0 = (v_0, 0, 0)$, $\boldsymbol{\omega}_0 = (0, 0, \omega_0)$!
d) Plot the trajectory of the center of mass of the ball for case c) for a few specific set of values of the parameters! How much do these trajectories differ from that of the pointlike object of charge Q and mass m, moving in the same magnetic field?

(Máté Vigh)

23. The faces of a planparallel plate are perpendicular to the x direction, and they are located at $x = 0$ and $x = \lambda_0$. There is an electromagnetic plane wave with wavelength λ_0 incident on the plane perpendicularly. The refraction index of the plane is dependent on x, and varies in the following way: $n(x) = 1 + x/\lambda_0$. Which is the fraction of the intensity transmitted by the plate? Determine the transfer matrix of the system!
Determine the transfer matrix for arbitrary $n(x)$ function (and plane waves propagating in the x direction).
Hint: Consider the plate as N very thin layers sliced perpendicularly to the x direction, and determine the transfer matrix for each layer; then take the limiting case $N \to \infty$!

(Márton Lájer)

24. If one wants to reach even the nearest stars within human time frame, spacecrafts of relativistic speeds are needed. A proposed method of propulsion is the antimatter engine: the craft carries matter-antimatter fuel as well as propellant matter that is accelerated with the energy released by annihilation.

Assume that we have an ideal engine, ie. the rate of propellant usage as well as the rate of the energy used for accelerating it (the power output of the engine) can be tuned freely and independently from each other, and the engine is always 100 percent efficient: the whole rest mass of the annihilating fuel is transformed into the kinetic energy of the propellant matter. However, we would like to minimize the antimatter usage (ie. we want to use the available antimatter as efficiently as possible). How should we control the engine (its power output and propellant usage) in order to achieve this?

Assume that the 'useful' mass of the spacecraft (ie. not including the propellant and the fuel) is 10000 metric tons, and with one refueling it can reach a speed of 200000 km/s, and also fully stop from this speed. How much antimatter is needed for this? How much propellant matter is needed? What should be the maximal power output of the engine, if the crew wants to feel the comfortable weight force corresponding to 1 g during the whole acceleration period? Compare these values to the contemporary total energy consumption of mankind!

(Márton Nagy)

25. Nordström's gravitational theory from 1912 assumes the gravitational field strength – similarly to the Newtonian case – to originate from the gradient of a scalar field Φ. The force acting on an object with rest mass M results in $F_k = M\,\partial_k\,\Phi$. Study within the framework of the theory the rules for the movement in static central potential, and calculate the trajectory of confined motion for the well known $\Phi(r) = -\alpha/r$ Kepler potential!

(Dávid Gyula)

26. The new theory of professor Zweistein contains a hypergravitational potential Φ as the source of the hypergravitational force, and in this case the force is inversely proportional to the rest mass M of the particle moving in this hyperpotential, $F_k = (K/M)\,\partial_k\,\Phi$, here K is a coupling constant with dimension of $(\text{mass})^2$. Let us study the movement of a particle of mass m which is at rest in the beginning (at $t = 0$), and let us try to follow the change of energy and momentum! Study in detail the case of homogeneous hypergravitational field ($\Phi = g\,z$, where g corresponds to the gravitational acceleration on the Earth surface). After how long time do the relativistic effects start to appear? What is the fate of the particle after a long time?

(Gyula Dávid)

27. A ring of radius R and of constant density ρ is fixed. Let us place a pointlike object with mass m on the line which is perpendicular to the circle's plane and pierces its middle, to the point where the gravitational force acting on the object is the highest!

Let the mass of the object be $m = 1$ kg, and the radius of the ring be $R = 1$ m, and the linear density is $\rho = 1\,\text{kg/m}$, whereas the gravitational constant is $\gamma = 6,67 \cdot 10^{-11}\text{Nm}^2/\text{kg}^2$.

What is the period of motion, if there is no other force present except for the gravitational force, and the object is at rest at the beginning of the cycle?

Let us try to determine the possible energy levels as precisely as possible using the Bohr–Sommerfeld quantization rules, in case of any values of the parameters ρ, m, γ, R!

(Gábor Homa)

28. Positronium is a 'hydrogen atom', in which instead of the proton a positron plays the role of the nucleus. Positronium really exists, however its components annihilate with each other after a short time in the range of nanoseconds. How large is the size of the positronium atom as compared to that of the hydrogen atom?

(János Major, co-founder of the Ortvay contest)

29. As it is well known from Douglas Adams (Hitchhiker's Guide to the Galaxy), Earth was produced for the purchase order of highly intelligent mice, or in other words the planet is a powerful computer, which is supposed to find the Final Question. To the Question, the Final Answer is 42 (and which is by the way the answer to 'the Life, the Universe and Everything'). The following physics fact is also illustrating the Answer. It is known that the eigenenergies of a three dimensional isotropic harmonic oscillator are determined by the n, l, m quantum numbers in the following way: $E_{n,l,m} = \hbar\omega(2n + l + \frac{3}{2})$. Determine the magic numbers of the harmonic oscillator!

Let us show, that twice the sum of the digits of the 42-nd magic number is 42!

(Gábor Homa)

30. Let us consider a very long, one dimensional linear chain consisting of identical atoms. (The number of atoms is N, where $N \to \infty$). Each atom contributes with one electron to the conduction band; all the other electrons are strongly bonded. Let us investigate the conduction electrons in tight binding approximation.
a) Consider first the (equilibrium) uniform case: the distance between neighboring atoms is a_0, the hopping integral is t_0. Determine the dispersion relation $\varepsilon_0(k)$ of the conduction electrons!
b) Then, consider the dimerized case, when the successive bond lengths regularly alternate. Let us describe this in the following manner: the odd and even numbered atoms shift left and right, respectively, from their equilibrium position with the same u value: $u_n = (-1)^n u$. Let us assume that (for small displacements) the hopping integral depends linearly on the bond length: $t = t_0 + \alpha(a - a_0)$. Determine the dispersion relation $\varepsilon(k)$ of the conduction electrons in this dimerized case!
c) Show that the total energy of the conduction electrons decreases in the dimerized state! How large is this decrease and how does it depend on u?
d) Consider the effect of the remaining electrons in elastic approximation: the elastic energy increases with Ku^2 due to the dimerization, with respect to the equilibrium energy. At which u^0 value is the total energy (elastic and the total energy of the conduction electrons) minimal?

(Jenő Kürti)

31. Let us place an electron in a tube whose axis is parallel to z and whose radius is R_0. There is a $V(z) = Dz^2/2$ harmonic potential acting on the electron. What will be the orbitals when a $\mathbf{B} = (B_r,\, B_\varphi,\, B_z) = (-\beta r/2,\, 0,\, B_0 + \beta z)$ magnetic field is applied (here (r, φ, z) denote the cylindrical polar coordinates)? (*Hint:* calculate exactly with the terms containing β and B_0 that do not break the separability of the Pauli equation, and consider the rest as perturbation.)
a) Define and calculate the spin–orbit coupling for this system.
b) How can we interpret the Stern–Gerlach experiment in terms of the above calculations? (Does it really prove that the component of the angular momentum or magnetic moment parallel to the magnetic field has to be quantized?)

(Titusz Fehér)

32. Examine the one dimensional scattering of a single massless fermion on a localized and rigid spin (neglect spin-flip processes). Such a system could be realized for example on the surface of a two dimensional topological insulator. Calculate the scattering matrix of the problem and compare your results with the scattering of a nonrelativistic particle under similar circumstances. Let the Hamiltonian for the two cases be

$$\hat{H}_1 = v\hat{p}_z\sigma_z + \mathbf{S}\boldsymbol{\sigma}\,\delta(z),$$
$$\hat{H}_2 = \frac{\hat{p}_z^2}{2m}\sigma_0 + \mathbf{S}\boldsymbol{\sigma}\,\delta(z),$$

where $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ is a vector formed from the Pauli matrices and σ_0 is the unit matrix.

(László Oroszlány)

33. Generalize the two-dimensional massless Dirac equation! Consider a particle moving on the (x, y) plane for which the Hamiltonian is $H = v\,\mathbf{Sp}$, where v is a parameter with the dimension of velocity, $\mathbf{p} = (p_x, p_y)$ is the momentum of the particle, $\mathbf{S} = (S_x, S_y, S_z)$ is the matrix representation of an arbitrary spin S (integer or half integer, ie, $S = \frac{1}{2}, 1, \frac{3}{2}, 2, \cdots$). Find the Landau levels $E_n(B)$ if the particle of charge Q is moving in a homogeneous magnetic field with $\mathbf{B} = (0, 0, B)$. What possible numbers can take the index n? Write a code to find the secular equation whose solutions give the Landau levels. Find the analytical expressions for the Landau levels in case of $S = \frac{1}{2}, 1, \frac{3}{2}, 2$. Show that the spectrum is symmetric for the energy $E = 0$. Show that if S is integer then there is an $E = 0$ Landau level, while for half integer S this level is missing.
Hint: Use the creation and annihilation operators.

(József Cserti)

34. Positronium is a ‘hydrogen atom’, in which instead of the proton a positron plays the role of the nucleus. Positronium really exists, however its components annihilate with each other after a short time in the range of nanoseconds. Many times the result of the annihilation process is a pair of photons with opposite momenta in the center-of-mass system. We perform an experiment in which a large number of positronium atoms fly in all directions of the space and we measure the energy distribution of the annihilation photons by an appropriate detector (this is generally a high-purity germanium crystal). Suppose that every detected photon originates from a two-photon annihilation process. How large is the minimum, the maximum, and the mean energy of the detected photons? What will be the form of the energy distribution?

(János Major, co-founder of the Ortvay contest)

35. Ancient greek atomists tried to reach down to the undivisible atom by cutting up a piece of cheese.

Let us design an experiment, in which a given amount of material is carved down to the smallest amount in a controllable fashion, such that the material loss during each cut is minimized!

The material is of free choice (except for illicit material), but the final result need to be well controllable.

The aim is to get to the piece of material which consists of the fewest number of atoms.

Let us suggest a set of rules, according to which the actual realizations can be evaluated!

(György Radnóczi Jr.)

36. A container of volume V with a hole of area A on its wall is filled with a dilute gas. Determine the proportion of particles escaping the container within a unit time if the velocity distribution of the particles is arbitrary. What is the result for particles of fixed velocity? How does the number of non-escaped particles change then in time?

(Tamás Tél)

37. The physicists of the Plasma Physics Institute of the Intergalactic Research Center find a gas bottle during the cleanup of the storage room. The identifier tag must have detached years ago, but it has been possible to find out that the bottle contains some isotope of hydrogen.

In finding the right isotope, the following measurement arrangement is designed: a controllable oven containing the unknown gas is heated up to a temperature where the hydrogen molecules break up and get ionized.

A cloud chamber is placed in homogeneous magnetic field, and attached to the oven such that the particles from the oven are emitted through a narrow window will end up in the cloud chamber.

Photographs are made recording the events in the cloud chamber, which are used to determine the radius of curvature of the particle trajectories.

Let us determine the ratio of the radii of the curvatures, provided that the gas bottle contains any of the hydrogen isotopes, protium (^{1}H), deuterium (^{2}H) or tritium (^{3}H).

Is there a possibility with this method to determine if the sample is exclusively a single isotope type, or a binary mixture? The temperature of the oven as well as the size of the magnetic field are unknown, but it is well reproducible for repeated measurements.

(Norbert Barankai)

38. The lightest way to break the Lorentz-invariance is to have slightly different limiting speeds for different particles:

$$E_s^2 = (p_s c_s)^2 + (m_s c_s^2)^2,$$

where c_s is now a constant parameter representative for each particle type (here the index s refers to the particle type) just like the m_s mass.

With the study of kinematics one can show that unusual processes will need to take place, or strange behaviour of usual processes would signal such a symmetry breaking scenario.

1) The neutral pion primarily decays to two photons, $\pi^0 \to 2\gamma$. Provided that $c_\pi < c_\gamma$, the photon instability will occur via the reaction $\gamma \to \pi^0 + \gamma$. Determine the threshold energy of this process (that is, the smallest energy where this is expected to take place) expressed with the small parameter δ representing the size of the breaking of the Lorentz-symmetry: $\delta \equiv (c_\gamma^2 - c_\pi^2)/c_\gamma^2 \ll 1$

2) The charged pions decay via the weak interaction process $\pi^+ \to \mu^+ + \nu_\mu$. This decay is energetically allowed in case of Lorentz-symmetry to hold. Study the decay kinematics of a relativistically high momentum pion ($p_\pi c_\pi \gg m_\pi c_\pi^2$) in the case of $c_\nu \neq c = c_\pi = c_\mu$.

Is there a threshold-effect, if $c_\nu > c$, that is, is there a lower limit for the energy of the fast pion? If the answer is yes, then let us express the limiting energy with the $\delta \equiv (c_\nu^2 - c^2)/2c_\nu^2 \ll 1$ parameter! How does the situation change if $c > c_\nu$?

(András Patkós)

39. The OPERA experiment situated in the Grans Sasso tunnel in Italy has reported that the neutrinos created at CERN (the distance between Grans Sasso and CERN is 730 km) arrive 60 ns earlier with respect to the one computed assuming the speed of light in vacuum. The anomaly corresponds to a relative difference of the neutrino velocity with respect to the speed of light 2.5×10^{-5}. Bertha Onestone knowing the general theory of her namesake believes, that the anomaly could be explained by the fact, that the neutrinos experience changing gravitational potential on their way to Italy inside the Earth. She just started to check the relevant formulae in the 2. book of the famous series in black (Landau: Theoretical Physics) when a Schwarzwald cake on her favorite cooking show grabbed her interest. Was she right? Is there such an effect, what is the estimated order of magnitude and how big is the correction?

(Gábor Cynolter)

40. An elderly person has a certain amount of savings and he is thinking about how to spend it in his remaining years. If he spends too fast, he will live his old days in poverty. If he spends too slow, and he happens to die soon, he has not used up the fruits of his active years. Let us formulate the dilemma as an optimization problem. One possible model is the following: Let our man be t_0 years old, with initial capital of M_0. Let the (random) year of his death be τ. By his chosen strategy, from t_0 till τ, he spends $S(t)$ in each year. Of course, $S(t)$ must always be smaller than his remaining capital $M(t)$. Thus the capital in the following year is $M(t+1) = M(t) - S(t)$. Let us define the integrated life enjoyment (U), to be maximized, as:

$$U = E\Big[\sum_{t=t_0}^{\tau} \ln \frac{S(t)}{A}\Big],$$

Here, $E[.]$ is the expectation over his random instance of death, A is a scale parameter, it is the salary above (below) which the life enjoyment is positive (negative).
a) What is the optimal $S(t)$ spending strategy? Plot $S(t)$ for people of different t_0 ages. Discuss the results.
Let us now assume our man buys a life annuity from a financial institution. He pays down his savings of M_0, in return the issuer pays him a fixed annual amount J (annuity), till his death. The annuity is priced such that the expected cashflow to the issuer is zero, plus a little margin is added.
b) Determine the annuity. Determine the integrated life enjoyment in this case. Is it beneficial to buy the life annuity? Discuss the result.
c) Propose extensions, modifications in the model, to study more general, or more realistic cases. Show results with at least one modification.
Use real life tables. Data for USA here:

`http://www.cdc.gov/nchs/products/life_tables.htm`

Data for different countries here:

`http://www.lifetable.de/cgi-bin/datamap.plx`

Let us neglect inflation (or imagine the capital is invested to counterbalance inflation, and we are calculating in terms of present value).

(Zsolt Bihary)

41. Dr. Absoluto Zero, the widely known philanthropic dictator of the small equatorial state Gummy Coast has created the world's first roller coaster which is perfectly straight. The Prime Physicist of the country (decorated by the National Coriolis Order for solving a former Ortvay problem) has studied for years the roller coasters in the western world, and realized that those ones going up and down and around are going out of fashion. People like those structures most where the car full of tourists is pulled up to a very high tower, then released towards the Earth (letting the passengers screaming pretty much) nearly at free fall. The rail turns later slowly to horizontal and the carriages are slowed down. „This is what we want, but much much bigger!" the dictator said , making his decision promptly. However, our people's straight and firm character is not compatible with letting the passage of the carriages diverted from a perfectly straight line – it must be straight forward, as my dear father, Absoluto Minus One has taught to me during hunting for gummy squirrel in the gummy woods. „I leave the details to you!" the dictator said to the Prime Physicist, who started out with the design.

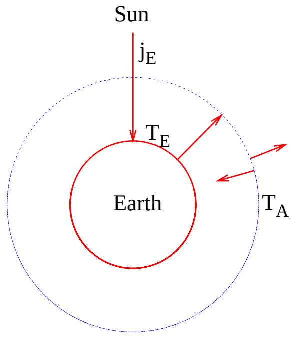
The enterprise drained rather dramatically the gross national product (especially the two space stations and the space suits for tourists in the national colours), but soon the perfectly straight roller coaster was constructed, with its main building in the main square of Port Goomy, capital of Gummy Coast. 'Straight to the future' – as the billboard announced.

The rails were perfectly frictionless and perfectly straight – they followed Eastwards and Westwards, slowly leaving the Earth surface and raised above the atmosphere up to the synchronous orbit, where they ended up on two special space stations. The passengers were pulled up to this point along the rail. After a momentary rest when the front end of the one hundred meter long train has been attached to the space station the carriages started free-fall towards the Earth – or more precisely, towards the main square of Port Goomy. (Fortunately, air resistance is negligible, as that, with all other forms of resistance, has been banned by Absoluto Zero some time earlier).

The train, after bursting through the capital's main square, ended up on the other space station (from where the passangers were released down on a ratline, purchasable for extra fare).
a) How long such a journey on the roller coaster lasted (from one of space stations to the other one)?
b) After the success of the world's first straight roller coaster, more models have been deployed (still perfectly straight ones) at different positions in the more temperate zones of Earth (the length of the line was the same as in Gummy Coast, but the space stations were not included in the project). Based on local political orientations, these were installed either along the East-West line, or along North-South. However, first attempts completing the journey revealed strange problems which were not experienced on the original installation on the equatorial Gummy Coast, and which prevented the people of those countries to fully enjoy the perfectly straight roller coasters. Which could possibly happen and why?

(Gyula Dávid)

42. Design your own climate model!



(0) As a zeroth-order approximation, determine the average surface temperature of the Earth (T_E) by assuming that, for thermodynamic purposes, the Earth behaves as a blackbody and the incoming and outgoing radiation fluxes are in equilibrium [the incoming energy flux from the Sun as measured by the satellites is equal to the so called solar constant ($j_E = 1.36 \cdot 10^3\,\mathrm{Js^{-1}m^{-2}}$)].

The result is worrysome (it would be too cold).

(1) In order to improve the situation, let us take into account the beneficial aspects of the greenhouse effect. Assume that the Earth is surrounded by a layer of greenhouse gas (blue circle in the attached figure) which is transparent to the radiation coming from the Sun (which is mainly high frequency radiation due to the high temperature of the Sun).

On the other hand, the radiation coming from the Earth (low frequency radiation since the temperature of the Earth is much lower than that of the Sun) is completely absorbed by this layer and then reradiated isotropically. From thermodynamics point of view, the greenhouse layer can be considered as a blackbody, and the temperature of this layer (T_A) as well as the temperature of the Earth (T_E) is again determined from the balance of the fluxes.

The result is worrisome again (it would be too hot).

(2) After so much failure, its time to introduce a model where we can calibrate the resulting temperature to the value we observe, $T_E = 15\,^\circ\mathrm{C}$. As a new element, enter the albedo (α) which determines what is the portion of the incoming radiation which is reflected in the high-frequency regime and thus becomes irrelevant in the energy balance of the Earth. Then the incoming energy flux can be written as

$$j_E(\mathrm{true}) = (1-\alpha) j_E\,.$$

The albedo depends on the temperature of the Earth, $\alpha = \alpha(T_E)$, since e.g. there are enormous ice fields during the ice ages which increase the albedo. At the other limit, at high temperatures, there is much evaporation and the increasing cloud cover may have a strong greenhouse effect which effectively decreases the albedo. Unfortunately, the clouds are not entirely understood since due to their reflectivity, they can increase the albedo as well. The answer to the question of which effect wins depends on what kind of cloud is formed and at what height they are formed. Thus the function $\alpha(T)$ is not really known.

Let us construct an $\alpha(T)$ function whose value at $T = 15\,^\circ\mathrm{C}$ leads to the present average temperature $T_E = 15\,^\circ\mathrm{C}$ of the Earth. Depending on the shape of the $\alpha(T)$ function, this temperature may be stable or unstable. Let us construct disaster scenarios where we freeze or get roasted.

Please, in no circumstances reveal your results to the press!

(Zoltán Rácz)

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